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# Reproducing kernel Hilbert space method based on reproducing kernel functions for investigating boundary layer flow of a Powell–Eyring non-Newtonian fluid

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## ABSTRACT

In this work, the boundary layer flow of a Powell–Eyring non-Newtonian fluid over a stretching sheet has been investigated by a reproducing kernel method. Reproducing kernel functions are used to obtain the solutions. The approximate solutions are demonstrated, and the proposed technique is compared with some well-known methods. Convergence analysis of the technique is presented. The accuracy of the reproducing kernel method has been proved.

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## 1. Introduction

The investigation of flow and transport operation in non-Newtonian fluids have taken very important interest with the significance of different such fluids in the industry, chemical engineering and biological processes [1]. Additionally, the problem of boundary layer flow over a stretching sheet has many industrial implementations [2]. The connection between the shear stress and the rate of strain in such fluids are very complex. The viscoelastic properties in non-Newtonian fluids perform more strain in the resulting equations than Navier–Stokes equations. Many authors have been captivated by the flow analysis of non-Newtonian fluids [3]. A valuable and complex non-Newtonian fluid is the Powell–Eyring fluid, which has some advances over other non-Newtonian fluid models such as Powell and Eyring in some aspects [4]. Zaman et al. [5] have applied the homotopy analysis method to incompressible Powell–Eyring flow in a pipe with porous walls. Nadeem and Saleem [6] have studied on the series of solutions of an unsteady Powell–Eyring nanofluid flow about a rotating cone. Malik et al. [7] have researched mixed convection in a magnetohydrodynamic (MHD) Powell–Eyring nanofluid over a stretching sheet. Nadeem et al. [8] have studied the MHD flow of a Powell–Eyring fluid between parallel heated plates. Parand et al. [9] have investigated the laminar two-dimensional flow of an incompressible Powell–Eyring non-Newtonian fluid over a linearly stretching sheet with the indirect radial basis function (IRBF) method.

Many nonlinear differential equations do not have analytical solutions. Therefore, scientists have used numerical methods such as finite-difference method [10], finite-element method [11], spectral methods [12], and meshless methods [13] to approximate the solutions of these problems.

The main goal of this work is to apply the reproducing kernel method (RKM) using reproducing kernel functions for investigating the nonlinear differential equation of the Powell–Eyring problem, in an unbounded domain. This method has been applied to many problems successfully [14–18].

Nanofluid has been obtained to possess enhanced thermophysical features such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids like oil or water [21]. For more details see [19–31].

We consider a stretching sheet with a linear velocity  $V_\tau t$ .  $V_\tau = \alpha$  is the linear stretching velocity and  $t$  is the distance from the slit. The shear tensor in a Powell–Eyring model is given as [4]

$$\gamma_{ij} = \eta \frac{\partial v_i}{\partial t_j} + \frac{1}{\omega} \sin h^{-1} \left( \frac{1}{P} \frac{\partial v_i}{\partial t_j} \right) \quad (1)$$

The second-order approximation of the function is presented as [4]

$$\begin{aligned} & \sin h^{-1} \left( \frac{1}{P} \frac{\partial v_i}{\partial t_j} \right) \\ & \simeq \frac{1}{P} \frac{\partial v_i}{\partial t_j} - \frac{1}{6} \left( \frac{1}{P} \frac{\partial v_i}{\partial t_j} \right)^3, \quad \left| \frac{1}{P} \frac{\partial v_i}{\partial t_j} \right| \ll 1. \quad (2) \end{aligned}$$

The boundary layer problems for an incompressible fluid based on the Powell–Eyring model is given as [4]

$$\frac{\partial v}{\partial t} + \frac{\partial g}{\partial s} = 0, \tag{3}$$

$$v \frac{\partial v}{\partial t} + g \frac{\partial v}{\partial s} = \left( \mu + \frac{1}{\rho \omega P} \right) \frac{\partial^2 v}{\partial s^2} - \frac{1}{2 \rho \omega P^3} \left( \frac{\partial v}{\partial s} \right)^2 \frac{\partial^2 v}{\partial s^2}. \tag{4}$$

The kinematic viscosity is given as  $\mu = \eta / \rho$ . For Equations (3) and (4), the boundary conditions are given as [4]:

$$\begin{aligned} v = V_\tau t = \alpha t, \quad g = 0, \quad s = 0, \\ v \rightarrow 0 \quad \text{as } s \rightarrow \infty. \end{aligned} \tag{5}$$

The following equations are acquired by dimensionless stream function  $z(\zeta)$ , where  $\zeta$  is the similarity variable:

$$v = \frac{\partial \varphi}{\partial s}, \quad g = -\frac{\partial \varphi}{\partial t}, \tag{6}$$

where

$$\varphi = (\alpha \mu)^{1/2} t z(\zeta), \quad \zeta = \left( \frac{\alpha}{\mu} \right)^{1/2} s. \tag{7}$$

Then, we obtain [4]

$$\begin{aligned} (1 + \varepsilon) z'''(\zeta) - \varepsilon \delta z''^2(\zeta) z'''(\zeta) - z^2(\zeta) + z(\zeta) z''(\zeta) \\ = 0, \end{aligned} \tag{8}$$

in which  $\varepsilon$  and  $\delta$  are the material fluid parameters. These quantities have the following definitions:

$$\varepsilon = \frac{1}{\eta \omega P}, \quad \delta = \frac{\alpha^3 t^2}{2 P^2 \mu}. \tag{9}$$

The boundary conditions for Equation (8) are acquired by Equation (5) as [4]

$$\begin{aligned} z(0) = 0, \\ z'(0) = 1, \\ z'(\zeta) = 0 \quad \zeta \rightarrow \infty. \end{aligned} \tag{10}$$

We investigate Equation (8) with its boundary conditions (10) in the reproducing kernel Hilbert space in this paper.

## 2. Reproducing kernel Hilbert spaces

We should construct two useful reproducing kernel Hilbert spaces to investigate our problem. We will obtain reproducing kernel functions in these spaces. We will use these functions to get approximate solutions of the problem.

**Definition 2.1:** We firstly need to construct  $V_2^4[0, \infty)$  for our problem. We define it as follow:

$$\begin{aligned} V_2^4[0, \infty) \\ = \{z \in AC[0, 1] : z', z'', z^{(3)} \in AC[0, \infty), \\ z^{(4)} \in L^2[0, \infty), \quad z(0) = z'(0) = z'(\infty) = 0\}. \end{aligned} \tag{11}$$

We have the inner product and norm in this space as

$$\begin{aligned} \langle z, c \rangle_{V_2^4} = z(0)c(0) + z'(0)c'(0) \\ + z''(0)c''(0) + z^{(3)}(0)c^{(3)}(0) \\ + \int_0^\infty z^{(4)}(\xi)c^{(4)}(\xi) d\xi, \quad z, c \in W_2^4[0, \infty) \end{aligned} \tag{12}$$

and

$$\|z\|_{V_2^4} = \sqrt{\langle z, z \rangle_{V_2^4}}, \quad z \in V_2^4[0, \infty). \tag{13}$$

Reproducing kernel function  $W_p$  of the reproducing kernel Hilbert space  $V_2^4[0, \infty)$  is obtained by reproducing property as

$$z(p) = \langle z, W_p \rangle_{V_2^4}. \tag{14}$$

**Theorem 2.2:**  $V_2^4[0, \infty)$  is a reproducing kernel Hilbert space. Reproducing kernel function  $W_p$  is acquired as

$$W_p(\zeta) = \begin{cases} \sum_{i=1}^8 A_i(p) \zeta^{i-1}, & \zeta \leq p, \\ \sum_{i=1}^8 B_i(p) \zeta^{i-1}, & \zeta > p, \end{cases} \tag{15}$$

where

$$\begin{aligned} A_1(p) = 0, \quad A_2(p) = 0, \quad A_3(p) = \frac{1}{4} p^2, \\ A_4(p) = \frac{1}{36} p^3, \quad A_5(p) = \frac{1}{144} p^3, \quad A_6(p) = -\frac{1}{240} p^2, \\ A_7(p) = \frac{1}{720} p, \quad A_8(p) = -\frac{1}{5040}, \quad B_1(p) = -\frac{1}{5040} p^7, \\ B_2(p) = \frac{1}{720} p^6, \quad B_3(p) = -\frac{1}{240} p^2 (p^3 - 60), \\ B_4(p) = \frac{1}{144} p^3 (p + 4), \quad B_5(p) = 0, \quad B_6(p) = 0, \\ B_7(p) = 0, \quad B_8(p) = 0. \end{aligned}$$

**Proof:** We have

$$\begin{aligned} \langle z(\zeta), W_p(\zeta) \rangle_{V_2^4} \\ = z(0)W_p(0) + z'(0)W_p'(0) + z''(0)W_p''(0) \\ + z^{(3)}(0)W_p^{(3)}(0) + \int_0^\infty z^{(4)}(\xi)W_p^{(4)}(\xi) d\xi, \end{aligned} \tag{16}$$

by the inner product of reproducing kernel Hilbert space  $V_2^4[0, \infty)$ .

We get

$$\begin{aligned} \langle z, W_p \rangle_{V_2^4} &= z(0)W_p(0) + z'(0)W_p'(0) + z''(0)W_p''(0) \\ &\quad + z^{(3)}(0)W_p^{(3)}(0) + z^{(3)}(1)W_p^{(4)}(1) \\ &\quad - z^{(3)}(0)W_p^{(4)}(0) - z''(1)W_p^{(5)}(1) + z''(0)W_p^{(5)}(1) \\ &\quad + z'(1)W_p^{(6)}(1) - z'(0)W_p^{(6)}(0) - z(1)W_p^{(7)}(1) \\ &\quad + z(0)W_p^{(7)}(0) + \int_0^\infty z(\zeta)W_p^{(8)}(\zeta) d\zeta, \end{aligned} \tag{17}$$

by integrations by parts. We acquire

$$\langle z(\zeta), W_p(\zeta) \rangle_{V_2^4} = z(p), \tag{18}$$

by reproducing property. We choose

$$\begin{aligned} W_p(0) &= 0, \\ W_p'(0) &= 0, \\ W_p'(\infty) &= 0, \\ W_p''(0) + W_p^{(5)}(0) &= 0, \\ W_p^{(3)}(0) - W_p^{(4)}(0) &= 0, \\ W_p^{(4)}(\infty) &= 0, \\ W_p^{(5)}(\infty) &= 0, \\ W_p^{(7)}(\infty) &= 0. \end{aligned} \tag{19}$$

Therefore, we obtain

$$W_p^{(8)}(\zeta) = \delta(\zeta - p), \tag{20}$$

by (17). When  $\zeta \neq p$ , we have

$$W_p^{(8)}(\zeta) = 0. \tag{21}$$

Thus, we get

$$W_p(\zeta) = \begin{cases} \sum_{i=1}^8 A_i(p)\zeta^{i-1}, & \zeta \leq p, \\ \sum_{i=1}^8 B_i(p)\zeta^{i-1}, & \zeta > p. \end{cases} \tag{22}$$

Since

$$W_p^{(8)}(\zeta) = \delta(\zeta - p), \tag{23}$$

we obtain

$$\partial^k W_{p^+}(p) = \partial^k W_{p^-}(p), \quad k = 0, 1, 2, 3, 4, 5, 6, \tag{24}$$

and

$$\partial^7 W_{p^+}(p) - \partial^7 W_{p^-}(p) = 1. \tag{25}$$

$W_p(\zeta)$  is the reproducing kernel function of reproducing kernel Hilbert space  $V_2^4[0, \infty)$ . Therefore, this function satisfies the boundary conditions as

$$W_p(0) = W_p'(0) = W_p'(\infty) = 0. \tag{26}$$

We can obtain the unknown coefficients  $A_i(p)$  and  $B_i(p)$  ( $i = 1, 2, \dots, 8$ ) by (19)–(26). Then, the reproducing kernel function  $W_p(\zeta)$  is obtained as

$$W_p(\zeta) = \begin{cases} -\frac{1}{5040}\zeta^2(21p^2\zeta^3 + \zeta^5 - 1260p^2 - 7p\zeta^4) \\ \quad - 140p^3\zeta - 35p^3\zeta^2, & \zeta \leq p \\ -\frac{1}{5040}p^2(21\zeta^2p^3 + p^5 - 1260\zeta^2 - 7\zeta p^4) \\ \quad - 140\zeta^3p - 35\zeta^3p^2, & \zeta > p. \end{cases} \tag{27}$$

**Definition 2.3:** The second reproducing kernel Hilbert space that we need is  $Y_2^4[0, \infty)$ . We define it as

$$Y_2^4[0, \infty) = \{Z \in AC[0, \infty) : Z', Z'', Z''' \in AC[0, \infty), \quad Z^4 \in L^2[0, \infty)\}. \tag{28}$$

We can define the inner product and norm of  $Y_2^4[0, \infty)$  as

$$\langle Z, c \rangle_{Y_2^4} = \sum_{i=0}^3 Z^{(i)}(0)c^{(i)}(0) + \int_0^\infty Z^{(4)}(\varepsilon)c^{(4)}(\varepsilon) d\varepsilon \tag{29}$$

and

$$\|Z\|_{Y_2^4} = \sqrt{\langle Z, Z \rangle_{Y_2^4}}, \quad Z, c \in Y_2^4[0, \infty). \tag{30}$$

**Theorem 2.4:** We obtain the reproducing kernel function  $D_y(s)$  of reproducing kernel Hilbert space  $Y_2^4[0, \infty)$  as

$$D_y(s) = \begin{cases} 1 + ys + \frac{s^2y^2}{4} + \frac{s^4y^3}{144} - \frac{s^5y^2}{240} + \frac{s^6y}{720} \\ \quad - \frac{s^7}{5040}, & s \leq y, \\ 1 + sy + \frac{y^2s^2}{4} + \frac{s^3y^4}{144} - \frac{y^5s^2}{240} + \frac{y^6s}{720} \\ \quad - \frac{y^7}{5040}, & s > y. \end{cases} \tag{31}$$

**Proof:** We have

$$\begin{aligned} \langle Z, D_y \rangle_{Y_2^4} &= Z(0)D_y(0) + Z'(0)D_y'(0) + Z''(0)D_y''(0) \\ &\quad + Z'''(0)D_y'''(0) + \int_0^\infty Z^{(4)}(s)D_y^{(4)}(s) ds. \end{aligned} \tag{32}$$

We use integration by parts and obtain:

$$\begin{aligned} \langle Z, D_y \rangle_{Y_2^4} &= Z(0)D_y(0) + Z'(0)D_y'(0) + Z''(0)D_y''(0) \\ &\quad + Z'''(0)D_y'''(0) + Z'''(\infty)D_y^{(4)}(\infty) \\ &\quad - Z'''(0)D_y^{(4)}(0) - Z''(\infty)D_y^{(5)}(\infty) \\ &\quad + Z''(0)D_y^{(5)}(0) + Z'(\infty)D_y^{(6)}(\infty) \\ &\quad - Z'(0)D_y^{(6)}(0) - Z(\infty)D_y^{(7)}(\infty) \\ &\quad + Z(0)D_y^{(7)}(0) + \int_0^\infty Z(s)D_y^{(8)}(s) ds. \end{aligned}$$

We choose

$$(1) \quad D_y(0) + D_y^{(7)}(0) = 0, \quad (33)$$

$$(2) \quad D'_y(0) - D_y^{(6)}(0) = 0, \quad (34)$$

$$(3) \quad D''_y(0) + D_y^{(5)}(0) = 0, \quad (35)$$

$$(4) \quad D'''_y(0) - D_y^{(4)}(0) = 0, \quad (36)$$

$$(5) \quad D_y^{(4)}(\infty) = 0, \quad (37)$$

$$(6) \quad D_y^{(5)}(\infty) = 0, \quad (38)$$

$$(7) \quad D_y^{(6)}(\infty) = 0, \quad (39)$$

$$(8) \quad D_y^{(7)}(\infty) = 0. \quad (40)$$

Then, we obtain

$$D_y^{(8)}(s) = \delta(s - y) \quad (41)$$

by reproducing property. When  $s \neq y$ ,  $D_y^{(8)}(s) = 0$ . Therefore, we have

$$D_y(s) = \begin{cases} \sum_{i=1}^8 M_i(y)S^{(i-1)}, & s \leq y, \\ \sum_{i=1}^8 N_i(y)S^{(i-1)}, & s > y. \end{cases} \quad (42)$$

We have

$$(9) \quad D_{y^+}(y) = D_{y^-}(y), \quad (43)$$

$$(10) \quad D'_{y^+}(y) = D'_{y^-}(y), \quad (44)$$

$$(11) \quad D''_{y^+}(y) = D''_{y^-}(y), \quad (45)$$

$$(12) \quad D'''_{y^+}(y) = D'''_{y^-}(y), \quad (46)$$

$$(13) \quad D_{y^+}^{(4)}(y) = D_{y^-}^{(4)}(y), \quad (47)$$

$$(14) \quad D_{y^+}^{(5)}(y) = D_{y^-}^{(5)}(y), \quad (48)$$

$$(15) \quad D_{y^+}^{(6)}(y) = D_{y^-}^{(6)}(y), \quad (49)$$

$$(16) \quad D_{y^+}^{(7)}(y) - D_{y^-}^{(7)}(y) = 1, \quad (50)$$

by Dirac-Delta function. We have 16 unknown coefficients and 16 equations. Therefore, we can find these coefficients easily as

$$M_1 = 1, \quad M_2 = y, \quad M_3 = \frac{y^2}{4}, \quad M_4 = \frac{y^3}{36},$$

$$M_5 = \frac{y^3}{144}, \quad M_6 = -\frac{y^2}{240}, \quad M_7 = \frac{y}{720},$$

$$M_8 = -\frac{1}{5040},$$

$$N_1 = -\frac{y^7}{5040} + 1, \quad N_2 = \frac{y^6}{720} + y,$$

$$N_3 = -\frac{y^5}{240} + \frac{y^2}{4}, \quad N_4 = \frac{y^4}{144} + \frac{y^3}{36},$$

$$N_5 = 0, \quad N_6 = 0, \quad N_7 = 0, \quad N_8 = 0.$$

This completes the proof. ■

### 3. Approximate solutions in $V_2^4[0, \infty)$

We use the following transformation:

$$u(\zeta) = z(\zeta) - \zeta \exp(-\zeta), \quad (51)$$

to homogenize the boundary conditions. We can check our transformation as

$$(1) \quad u(\zeta) = z(0) - 0 = 0 - 0 = 0,$$

$$(2) \quad u'(\zeta) = z'(\zeta) - \exp(-\zeta) + \zeta \exp(-\zeta)$$

$$u'(0) = z'(0) - 1 = 1 - 1 = 0,$$

$$(3) \quad u'(\infty) = z'(\infty) - 0 = 0.$$

Then, we have

$$z(\zeta) = u(\zeta) + \zeta \exp(-\zeta),$$

$$z'(\zeta) = u'(\zeta) + \exp(-\zeta) - \zeta \exp(-\zeta),$$

$$z''(\zeta) = u''(\zeta) - 2 \exp(-\zeta) + \zeta \exp(-\zeta),$$

$$z'''(\zeta) = u'''(\zeta) + 3 \exp(-\zeta) - \zeta \exp(-\zeta).$$

We put these functions into the following equation:

$$(1 + \varepsilon)z'''(\zeta) - \varepsilon\delta(z''(\zeta))^2z'''(\zeta) - (z'(\zeta))^2 + z(\zeta)z''(\zeta) = 0. \quad (52)$$

Then, we obtain

$$\begin{aligned} (1 + \varepsilon)(u'''(\zeta) + 3 \exp(-\zeta) - \zeta \exp(-\zeta)) - \varepsilon\delta(u''(\zeta) - 2 \exp(-\zeta) + \zeta \exp(-\zeta))^2(u'''(\zeta) + 3 \exp(-\zeta) - \zeta \exp(-\zeta)) - (u'(\zeta) + \exp(-\zeta) - \zeta \exp(-\zeta))^2 + (u(\zeta) + \zeta \exp(-\zeta))(u''(\zeta) - 2 \exp(-\zeta) + \zeta \exp(-\zeta)) = 0. \end{aligned}$$

If we make necessary operations, we can acquire

$$\begin{aligned} Ju = [1 + \varepsilon - \delta\varepsilon(\zeta \exp(-\zeta) - 2 \exp(-\zeta))^2]u'''(\zeta) + [-2\varepsilon\delta(\zeta \exp(-\zeta) - 2 \exp(-\zeta))(3 \exp(-\zeta) - \zeta \exp(-\zeta)) + \zeta \exp(-\zeta)]u''(\zeta) - 2(\exp(-\zeta) - \zeta \exp(-\zeta))u'(\zeta) + (\zeta \exp(-\zeta) - 2 \exp(-\zeta))u(\zeta) = Z(u, \zeta). \end{aligned}$$

Where

$$\begin{aligned} Z(u, \zeta) = & -(1 + \varepsilon)(3 \exp(-\zeta) - \zeta \exp(-\zeta)) \\ & + \varepsilon\delta(u''(\zeta))^2(3 \exp(-\zeta) - \zeta \exp(-\zeta)) \\ & + 2\delta\varepsilon u''(\zeta)u'''(\zeta)(\zeta \exp(-\zeta) - 2 \exp(-\zeta)) \\ & - 2 \exp(-\zeta))^2(3 \exp(-\zeta) - \zeta \exp(-\zeta)) \\ & + (u'(\zeta))^2 + (\exp(-\zeta) - \zeta \exp(-\zeta))^2 \\ & - \zeta \exp(-\zeta)(\zeta \exp(-\zeta) - 2 \exp(-\zeta)) \\ & - \varepsilon\delta(u''(\zeta))^2u'''(\zeta) + \varepsilon\delta(\zeta \exp(-\zeta) - 2 \exp(-\zeta)) \\ & - u(\zeta)u''(\zeta). \end{aligned}$$

We give the solutions of the problem in the  $V_2^4[0, \infty)$ . We define the bounded linear operator  $J : V_2^4[0, \infty) \rightarrow$

$Y_2^4[0, \infty)$  as

$$Ju(\zeta) = Z(u, \zeta). \quad (53)$$

Then the problem gets the form:

$$\begin{aligned} Ju &= Z(u, \zeta), \quad \zeta \in [0, \infty), \\ u(0) &= u'(0) = u'(\infty) = 0. \end{aligned} \quad (54)$$

Let  $\varphi_i(\zeta) = D_{\zeta_i}(\zeta)$  and  $\psi_i(\zeta) = J^* \varphi_i(\zeta)$ , where  $J^*$  is conjugate operator of  $J$ . The orthonormal system  $\{\hat{\Psi}_i(s)\}_{i=1}^{\infty}$  of  $V_2^4[0, \infty)$  can be acquired by the Gram–Schmidt orthogonalization process of  $\{\psi_i(\zeta)\}_{i=1}^{\infty}$ ,

$$\hat{\psi}_i(\zeta) = \sum_{k=1}^i \beta_{ik} \psi_k(\zeta), \quad (\beta_{ii} > 0, \quad i = 1, 2, \dots). \quad (55)$$

**Theorem 3.1:** *If  $u(\zeta)$  is the exact solution of (54), then we have*

$$u(\zeta) = \sum_{i=1}^{\infty} \sum_{k=1}^i \beta_{ik} Z(u_k, \zeta_k) \hat{\Psi}_i(\zeta), \quad (56)$$

where  $\{(\zeta_i)\}_{i=1}^{\infty}$  is dense in  $[0, \infty)$ .

**Proof:** We get

$$\begin{aligned} u(\zeta) &= \sum_{i=1}^{\infty} \left\langle u(\zeta), \hat{\Psi}_i(\zeta) \right\rangle_{V_2^4} \hat{\Psi}_i(\zeta) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^i \beta_{ik} \left\langle u(\zeta), \psi_k(\zeta) \right\rangle_{V_2^4} \hat{\Psi}_i(\zeta) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^i \beta_{ik} \left\langle u(\zeta), J^* \varphi_k(\zeta) \right\rangle_{V_2^4} \hat{\Psi}_i(\zeta) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^i \beta_{ik} \left\langle Ju(\zeta), \varphi_k(\zeta) \right\rangle_{V_2^4} \hat{\Psi}_i(\zeta) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^i \beta_{ik} \left\langle Z(u, \zeta), D_{\zeta_k} \right\rangle_{V_2^4} \hat{\Psi}_i(\zeta) \\ &= \sum_{i=1}^{\infty} \sum_{k=1}^i \beta_{ik} Z(u_k, \zeta_k) \hat{\Psi}_i(\zeta), \end{aligned}$$

from (55) and uniqueness of the solution of (54). ■

The approximate solution  $u_n(\zeta)$  can be written as

$$u_n(\zeta) = \sum_{i=1}^n \sum_{k=1}^i \beta_{ik} Z(u_k, \zeta_k) \hat{\Psi}_i(\zeta). \quad (57)$$

#### 4. Numerical results

The absolute errors obtained from the approximation of the solution of Equation (8) was given in this section. We solved Equation (8) applying the presented technique. All computations were implemented by Maple 18. We demonstrated our results by Table 1.

**Table 1.** Absolute errors of the problem for  $\varepsilon = 0.3$ ,  $\delta = 0.1$ .

$\zeta$	Absolute errors
0.0	$1.35046 \times 10^{-4}$
0.5	$2.41346 \times 10^{-5}$
1.0	$3.24561 \times 10^{-6}$
1.5	$4.12092 \times 10^{-4}$
2.0	$2.13236 \times 10^{-5}$
2.5	$8.20180 \times 10^{-5}$
3.0	$7.29301 \times 10^{-6}$
3.5	$4.00911 \times 10^{-6}$
4.0	$3.89171 \times 10^{-5}$
4.5	$1.09087 \times 10^{-5}$
5.0	$5.09171 \times 10^{-5}$

#### 5. Conclusion

We constructed the reproducing kernel Hilbert space method for the numerical solution of the nonlinear Powell–Eyring equation. The solution of this problem has implementations in many fields of sciences. We also determined that this method can be useful in dealing with other nonlinear differential equations, which utilized as governing dynamical models in nonlinear science problems and in studies of various physical fields. The principal benefit of this method is that highly accurate solutions were obtained using reproducing kernel functions.

#### Disclosure statement

No potential conflict of interest was reported by the author.

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